Review of Literature on SINDy and its Relation to Bipolar Disorder

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Bipolar Disorder

Bipolar disorder is a mood disorder in which patients suffer from cycles of depressive and hypo-manic episodes. There are 2 main variations of this disorder, BP I and BP II. These variations share common traits such as cycles of depressive and manic episodes[1, 8, 7]. A key distinction of BP II is rapid cycling, or having 4 or more depression-mania cycles within one year [8, 2]. Due to this cycling between two emotional states, BP II is well suited for modeling by limit cycle oscillations.

Proper modeling for BP II has the potential to enhance our understanding of the disorder, leading to improved diagnosis and treatment methods. BP II affects approximately 1% of the US adult population, but proper diagnosis and treatment is difficult. This difficulty leads to common cases of misdiagnosis, with patients being frequently diagnosed as having unipolar depression or severe personality disorder. [2, 6]. In some cases, proper diagnosis can take as long as 10 years.

Treatment of BP II is also difficult, as patients often do not adhere to treatment plans due to their shifting mental state. These treatments typically rely on individually toxic drugs, which leads to multi drug treatments meant to alleviate prescription symptoms. [5, 2]. Many of the drugs are antidepressants, which take several days or weeks to reach the proper levels within the blood stream. Finding the proper treatment for patients can take several months or years of trial and error, and takes even longer if incorrectly diagnosed [6, 2].

Using Differential Equations to Model Bipolar II Disorder Cycles

The specific equations we tested were based on previous work done by [2]. Following their process, the models we tested were a negatively damped harmonic oscillator (1) and a Van der Pol oscillator (2). We'll refer to these models as our linear and nonlinear models, respectively.

$$x'' - \alpha x' + \omega^2 x = 0 \tag{1}$$

$$x'' - \alpha x' + \omega^2 x - \beta x^2 x' = 0 \tag{2}$$

These equations represent the behavior of an untreated individual diagnosed with BP II disorder. Here, x represents the emotional state of the individual and x' represents the change in emotional state over time. We set $\alpha = 0.36$, $\beta = -100$ and $\omega = 5$, which matches the parameters used by [2].

Of our two equations, we expected the nonlinear model produce a more realistic representation of BP II behavior. The linear model increases infinitely as x increases, but the severity of an individual's symptoms cannot realistically do the same. We will still test the linear model to see whether or not it is a viable base equation for modeling cyclic behaviors at all. However, a linear model is itself not a good candidate for a cyclic behavioral model due to its infinitely increasing nature. The nonlinear model is a better candidate since it is simply a modified linear equation, with an additional forcing factor that prevents it from increasing infinitely.

The SINDy Algorithm

SINDy, or Sparse Identification of Nonlinear Dynamical Systems, is a method designed to analyze mass amounts of data to find the governing equations. This process begins with gathering time-series data, which is then used to create a library of potential candidate functions, such as polynomial and trigonometric terms [3]. The algorithm identifies the most influential terms by implementing regression methods such as LASSO regression, allowing for the interactions of interest to be described in simpler mathematical terms [4, 3].

After identifying the governing equations, SINDy provides a means to analyze and predict the system's behavior across different scenarios. The resulting models can be interpreted, allowing for valuable insights into the underlying dynamics that may not easily appear in raw datasets to become visible [4, 3]. This approach is especially beneficial in areas such as genomics, finance, and robotics where grasping the governing processes is essential.

Simulating the Data

In order to produce the simulated data sets, our equations needed to be in the form of a system of differential equations. The linear model (1) and nonlinear model (2) decomposed into the following systems:

$$\begin{aligned} x_0 &= x' \\ x_0' &= \alpha x' - \omega^2 x \end{aligned} \tag{3}$$

$$\begin{aligned} x_0 &= x' \\ x_0' &= x'(\alpha + \beta x^2) - \omega^2 x \end{aligned} \tag{4}$$

Results

SINDy was able to successfully discover governing equations for each data set we used. For the linear model (3), SINDy returned system (5). The returned coefficients were nearly identical to the ones in the original system, so these results are encouraging. The results of the nonlinear model (4) was system (6).

Results Cont.

$$\begin{aligned} x_0 &= x' \\ x_0' &= -24.990x + 0.360x' \end{aligned} \tag{5}$$

$$\begin{aligned} x_0 &= x' \\ x_0' &= 0.272 - 12.944x - 0.211x' + 5.477x^2 - 22.971xx' - 0.838(x')^2 \end{aligned} \tag{6}$$

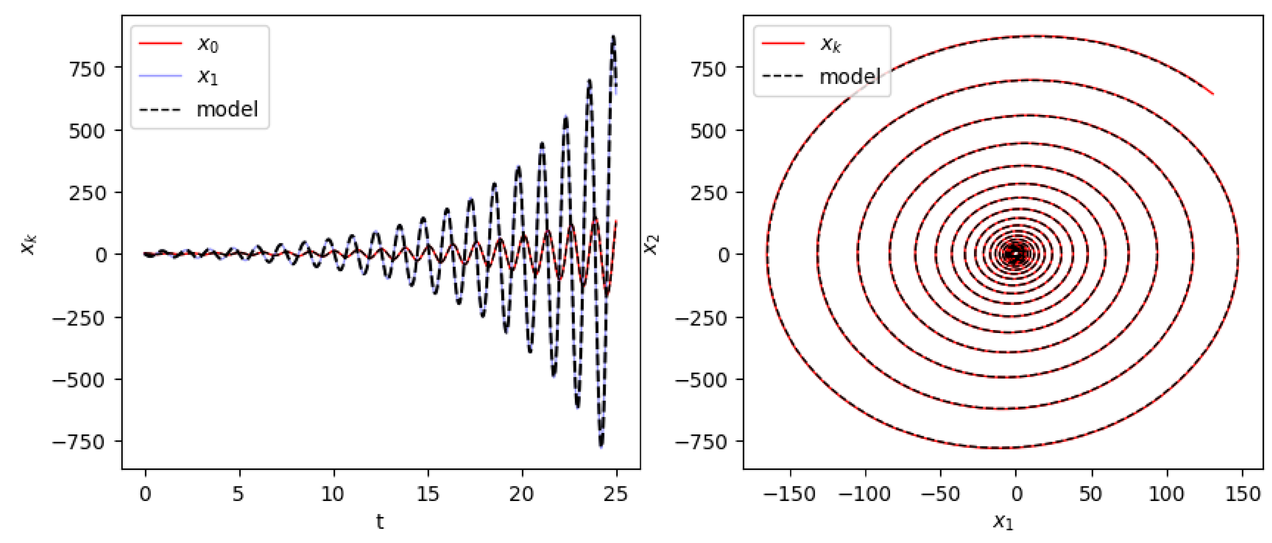

Figure 1. SINDy Algorithm Output for the Negatively Damped Oscillator Model

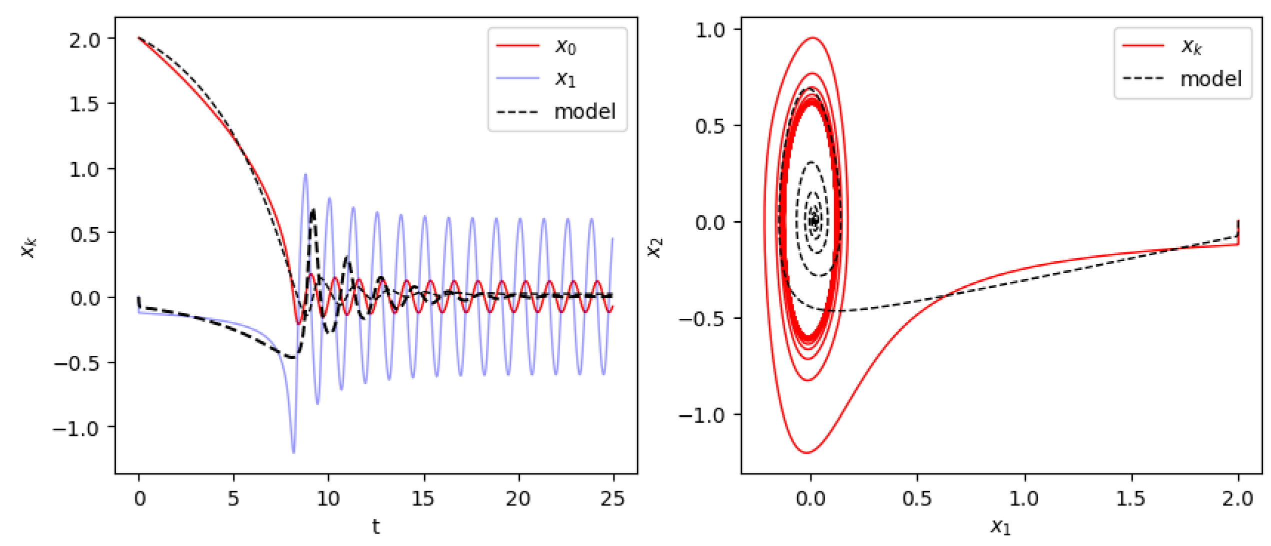

Figure 2. SINDy Algorithm Output for the Van der Pol Equation Model

Discussion

After generating training data using the linear model found by [2], SINDy was able to correctly return the variables of the governing equation with minimal variations from the coefficients that we provided. The linear model is a good starting point when modeling untreated BP II disorder, as it effectively captures the increase in mood severity over time. However, the model does not contain a limit, but instead the size of the oscillations will continue to grow towards infinity over time.

Using the same process, we generated data using the nonlinear model found by [2]. While SINDy was not quite able to reproduce the correct governing equation, it was able to produce something similar. The equation SINDy produced did not contain the term x^2x' which was featured in the governing equation, instead replacing the single missing term with four new terms: a constant term, x^2, xx', and $(x')^2$.

This equation presented by SINDy highlights one of the possible downfalls of SINDy, and that is possible to get incorrect conclusions. SINDy generated a function that behaves similarly to the function governing the data, but of a different form. This is due to an incorrect sparsity vector, possibly caused by too small of a training dataset.

Conclusion

In this paper, we generated data using the two theoretical governing equations for the emotional state of an untreated BP II individual. Each dataset was then analyzed using the SINDy algorithm, which returned an equation that models the data that we generated. SINDy was able to very closely reproduce our linear model, but was not able to reproduce our nonlinear one quite as closely. In general, SINDy could be used in the future to generate a specific oscillating function for each BP II patient, allowing doctors to assign treatment to patients that takes their emotional cycle into account instead of using a general model that might not meet all patients' needs.

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